

**A new National Curriculum for Mathematics was introduced for all years at Robert Bloomfield in September 2015. Below are extracts from the actual document.**

### **Purpose of Study**

Mathematics is a creative and highly inter-connected discipline that has been developed over centuries, providing the solution to some of history's most intriguing problems. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject.

### **Aims**

The national curriculum for mathematics aims to ensure that all pupils:

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
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- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. The programmes of study are, by necessity, organised into apparently distinct domains, but pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge to science and other subjects.

The expectation is that the majority of pupils will move through the programmes of study at broadly the same pace. However, decisions about when to progress should always be based on the security of pupils' understanding and their readiness to progress to the next stage. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content. Those who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practice, before moving on.

### **Information and Communication Technology (ICT)**

Calculators should not be used as a substitute for good written and mental arithmetic. They should therefore only be introduced near the end of key stage 2 to support pupils' conceptual understanding and exploration of more complex number problems, if written and mental arithmetic are secure. Teachers should use their judgement about when ICT tools should be used.

### **Spoken Language**

The national curriculum for mathematics reflects the importance of spoken language in pupils' development across the whole curriculum – cognitively, socially and linguistically. The quality and variety of language that pupils hear and speak are key factors in developing their mathematical vocabulary and presenting a mathematical justification, argument or proof. They must be assisted in making their thinking clear to themselves as well as others and teachers should ensure that pupils build secure foundations by using discussion to probe and remedy their misconceptions.

## **School Curriculum**

The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study.

## **Attainment Targets**

By the end of each key stage, pupils are expected to know, apply and understand the matters, skills and processes specified in the relevant programme of study.

## **Key Stage 1 (Year 1 & 2)**

The principal focus of mathematics teaching in key stage 1 is to ensure that pupils develop confidence and mental fluency with whole numbers, counting and place value. This should involve working with numerals, words and the four operations, including with practical resources [for example, concrete objects and measuring tools].

At this stage, pupils should develop their ability to recognise, describe, draw, compare and sort different shapes and use the related vocabulary. Teaching should also involve using a range of measures to describe and compare different quantities such as length, mass, capacity/volume, time and money.

By the end of year 2, pupils should know the number bonds to 20 and be precise in using and understanding place value. An emphasis on practice at this early stage will aid fluency.

Pupils should read and spell mathematical vocabulary, at a level consistent with their increasing word reading and spelling knowledge at key stage 1.

## **Year 1 Programme of Study**

### **Number – Number and Place Value**

Pupils should be taught to:

- count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number
- count, read and write numbers to 100 in numerals; count in multiples of twos, fives and tens
- given a number, identify one more and one less
- identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least
- read and write numbers from 1 to 20 in numerals and words.

### **Number – Addition and Subtraction**

Pupils should be taught to:

- read, write and interpret mathematical statements involving addition (+), subtraction (−) and equals (=) signs
- represent and use number bonds and related subtraction facts within 20
- add and subtract one-digit and two-digit numbers to 20, including zero
- solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as  $7 = \square - 9$ .

## **Number – Multiplication and Division**

Pupils should be taught to:

- solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

## **Number – Fractions**

Pupils should be taught to:

- recognise, find and name a half as one of two equal parts of an object, shape or quantity
- recognise, find and name a quarter as one of four equal parts of an object, shape or quantity.

## **Measurement**

Pupils should be taught to:

- compare, describe and solve practical problems for:
- lengths and heights [for example, long/short, longer/shorter, tall/short, double/half]
- mass/weight [for example, heavy/light, heavier than, lighter than]
- capacity and volume [for example, full/empty, more than, less than, half, half full, quarter]
- time [for example, quicker, slower, earlier, later]
- measure and begin to record the following:
- lengths and heights
- mass/weight
- capacity and volume
- time (hours, minutes, seconds)
- recognise and know the value of different denominations of coins and notes
- sequence events in chronological order using language [for example, before and after, next, first, today, yesterday, tomorrow, morning, afternoon and evening]
- recognise and use language relating to dates, including days of the week, weeks, months and years
- tell the time to the hour and half past the hour and draw the hands on a clock face to show these times.

## **Geometry – Properties of Shapes**

Pupils should be taught to:

- recognise and name common 2-D and 3-D shapes, including:
- 2-D shapes [for example, rectangles (including squares), circles and triangles]
- 3-D shapes [for example, cuboids (including cubes), pyramids and spheres].

## **Geometry – Position and Direction**

Pupils should be taught to:

- describe position, direction and movement, including whole, half, quarter and three-quarter turns.

## Year 2 Programme of Study

### Number – Number and Place Value

Pupils should be taught to:

- count in steps of 2, 3, and 5 from 0, and in tens from any number, forward and backward
- recognise the place value of each digit in a two-digit number (tens, ones)
- identify, represent and estimate numbers using different representations, including the number line
- compare and order numbers from 0 up to 100; use  $<$ ,  $>$  and  $=$  signs
- read and write numbers to at least 100 in numerals and in words
- use place value and number facts to solve problems.

### Number – Addition and Subtraction

Pupils should be taught to:

- solve problems with addition and subtraction:
- using concrete objects and pictorial representations, including those involving numbers, quantities and measures
- applying their increasing knowledge of mental and written methods
- recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
- add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
  - a two-digit number and ones
  - a two-digit number and tens
  - two two-digit numbers
- adding three one-digit numbers
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

### Number – Multiplication and Division

Pupils should be taught to:

- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication ( $\times$ ), division ( $\div$ ) and equals ( $=$ ) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

### Number – Fractions

Pupils should be taught to:

- recognise, find, name and write fractions  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{2}{4}$  and  $\frac{3}{4}$  of a length, shape, set of objects or quantity
- write simple fractions for example,  $\frac{1}{2}$  of 6 = 3 and recognise the equivalence of  $\frac{1}{2}$  and  $\frac{2}{4}$

## Measurement

Pupils should be taught to:

- choose and use appropriate standard units to estimate and measure length/height in any direction (m/cm); mass (kg/g); temperature ( $^{\circ}\text{C}$ ); capacity (litres/ml) to the nearest appropriate unit, using rulers, scales, thermometers and measuring vessels
- compare and order lengths, mass, volume/capacity and record the results using  $>$ ,  $<$  and  $=$
- recognise and use symbols for pounds (£) and pence (p); combine amounts to make a particular value
- find different combinations of coins that equal the same amounts of money
- solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change
- compare and sequence intervals of time
- tell and write the time to five minutes, including quarter past/to the hour and draw the hands on a clock face to show these times
- know the number of minutes in an hour and the number of hours in a day.

## Geometry – Properties of Shapes

Pupils should be taught to:

- identify and describe the properties of 2-D shapes, including the number of sides and line symmetry in a vertical line
- identify and describe the properties of 3-D shapes, including the number of edges, vertices and faces
- identify 2-D shapes on the surface of 3-D shapes, [for example, a circle on a cylinder and a triangle on a pyramid]
- compare and sort common 2-D and 3-D shapes and everyday objects.

## Geometry – Position and Direction

Pupils should be taught to:

- order and arrange combinations of mathematical objects in patterns and sequences
- use mathematical vocabulary to describe position, direction and movement, including movement in a straight line and distinguishing between rotation as a turn and in terms of right angles for quarter, half and three-quarter turns (clockwise and anti-clockwise).

## Statistics

Pupils should be taught to:

- interpret and construct simple pictograms, tally charts, block diagrams and simple tables
- ask and answer simple questions by counting the number of objects in each category and sorting the categories by quantity
- ask and answer questions about totalling and comparing categorical data.

## Lower Key Stage 2 – Years 3 and 4

The principal focus of mathematics teaching in lower key stage 2 is to ensure that pupils become increasingly fluent with whole numbers and the four operations, including number facts and the concept of place value. This should ensure that pupils develop efficient written and mental methods and perform calculations accurately with increasingly large whole numbers.

At this stage, pupils should develop their ability to solve a range of problems, including with simple fractions and decimal place value. Teaching should also ensure that pupils draw with increasing accuracy and develop mathematical reasoning so they can analyse shapes and their properties, and confidently describe the relationships between them. It should ensure that they can use measuring instruments with accuracy and make connections between measure and number.

By the end of year 4, pupils should have memorised their multiplication tables up to and including the 12 multiplication table and show precision and fluency in their work.

Pupils should read and spell mathematical vocabulary correctly and confidently, using their growing word reading knowledge and their knowledge of spelling.

## Year 3 Programme of Study

### **Number – Number and Place Value**

Pupils should be taught to:

- count from 0 in multiples of 4, 8, 50 and 100; find 10 or 100 more or less than a given number
- recognise the place value of each digit in a three-digit number (hundreds, tens, ones)
- compare and order numbers up to 1000
- identify, represent and estimate numbers using different representations
- read and write numbers up to 1000 in numerals and in words
- solve number problems and practical problems involving these ideas.

### **Number – Addition and Subtraction**

Pupils should be taught to:

- add and subtract numbers mentally, including:
  - a three-digit number and ones
  - a three-digit number and tens
  - a three-digit number and hundreds
- add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

### **Number – Multiplication and Division**

Pupils should be taught to:

- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which  $n$  objects are connected to  $m$  objects.

## Number – Fractions

Pupils should be taught to:

- count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10
- recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators
- recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators
- recognise and show, using diagrams, equivalent fractions with small denominators
- add and subtract fractions with the same denominator within one whole [for example,  $\frac{5}{7} + \frac{1}{7} = \frac{6}{7}$ ]
- compare and order unit fractions, and fractions with the same denominators
- solve problems that involve all of the above.

## Measurement

Pupils should be taught to:

- measure, compare, add and subtract: lengths (m/cm/mm); mass (kg/g); volume/capacity (l/ml)
- measure the perimeter of simple 2-D shapes
- add and subtract amounts of money to give change, using both £ and p in practical contexts
- tell and write the time from an analogue clock, including using Roman numerals from I to XII, and 12-hour and 24-hour clocks
- estimate and read time with increasing accuracy to the nearest minute; record and compare time in terms of seconds, minutes and hours; use vocabulary such as o'clock, a.m./p.m., morning, afternoon, noon and midnight
- know the number of seconds in a minute and the number of days in each month, year and leap year
- compare durations of events [for example to calculate the time taken by particular events or tasks].

## Geometry – Properties of Shapes

Pupils should be taught to:

- draw 2-D shapes and make 3-D shapes using modelling materials; recognise 3-D shapes in different orientations and describe them
- recognise angles as a property of shape or a description of a turn
- identify right angles, recognise that two right angles make a half-turn, three make three quarters of a turn and four a complete turn; identify whether angles are greater than or less than a right angle
- identify horizontal and vertical lines and pairs of perpendicular and parallel lines.

## Statistics

Pupils should be taught to:

- interpret and present data using bar charts, pictograms and tables
- solve one-step and two-step questions [for example, 'How many more?' and 'How many fewer?'] using information presented in scaled bar charts and pictograms and tables.

## Year 4 Programme of Study

### Number – Number and Place Value

Pupils should be taught to

- count in multiples of 6, 7, 9, 25 and 1000
- find 1000 more or less than a given number
- count backwards through zero to include negative numbers
- recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)
- order and compare numbers beyond 1000
- identify, represent and estimate numbers using different representations
- round any number to the nearest 10, 100 or 1000
- solve number and practical problems that involve all of the above and with increasingly large positive numbers
- read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value.

### Number – Addition and Subtraction

Pupils should be taught to:

- add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.

### Number – Multiplication and Division

Pupils should be taught to:

- recall multiplication and division facts for multiplication tables up to  $12 \times 12$
- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as  $n$  objects are connected to  $m$  objects.

### Number – Fractions (including decimals)

Pupils should be taught to:

- recognise and show, using diagrams, families of common equivalent fractions
- count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten.
- solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number
- add and subtract fractions with the same denominator
- recognise and write decimal equivalents of any number of tenths or hundredths
- recognise and write decimal equivalents to  $\frac{1}{4}$ ,  $\frac{1}{2}$ ,  $\frac{3}{4}$

- find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths
- round decimals with one decimal place to the nearest whole number
- compare numbers with the same number of decimal places up to two decimal places
- solve simple measure and money problems involving fractions and decimals to two decimal places.

## Measurement

Pupils should be taught to:

- convert between different units of measure [for example, kilometre to metre; hour to minute]
- measure and calculate the perimeter of a rectilinear figure (including squares) in centimetres and metres
- find the area of rectilinear shapes by counting squares
- estimate, compare and calculate different measures, including money in pounds and pence
- read, write and convert time between analogue and digital 12- and 24-hour clocks
- solve problems involving converting from hours to minutes; minutes to seconds; years to months; weeks to days.

## Geometry – Properties of Shapes

Pupils should be taught to:

- compare and classify geometric shapes, including quadrilaterals and triangles, based on their properties and sizes
- identify acute and obtuse angles and compare and order angles up to two right angles by size
- identify lines of symmetry in 2-D shapes presented in different orientations
- complete a simple symmetric figure with respect to a specific line of symmetry.

## Geometry – Position and Direction

Pupils should be taught to:

- describe positions on a 2-D grid as coordinates in the first quadrant
- describe movements between positions as translations of a given unit to the left/right and up/down
- plot specified points and draw sides to complete a given polygon.

## Statistics

Pupils should be taught to:

- interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graphs.
- solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and other graphs.

## Upper Key Stage 2 – Years 5 and 6

The principal focus of mathematics teaching in upper key stage 2 is to ensure that pupils extend their understanding of the number system and place value to include larger integers. This should develop the connections that pupils make between multiplication and division with fractions, decimals, percentages and ratio.

At this stage, pupils should develop their ability to solve a wider range of problems, including increasingly complex properties of numbers and arithmetic, and problems demanding efficient written and mental

methods of calculation. With this foundation in arithmetic, pupils are introduced to the language of algebra as a means for solving a variety of problems. Teaching in geometry and measures should consolidate and extend knowledge developed in number. Teaching should also ensure that pupils classify shapes with increasingly complex geometric properties and that they learn the vocabulary they need to describe them.

By the end of year 6, pupils should be fluent in written methods for all four operations, including long multiplication and division, and in working with fractions, decimals and percentages.

Pupils should read, spell and pronounce mathematical vocabulary correctly.

## **Year 5 Programme of Study**

### **Number – Number and Place Value**

Pupils should be taught to:

- read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit
- count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000
- interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through zero
- round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000
- solve number problems and practical problems that involve all of the above
- read Roman numerals to 1000 (M) and recognise years written in Roman numerals.

### **Number – Addition and Subtraction**

Pupils should be taught to:

- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- add and subtract numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

### **Number – Multiplication and Division**

Pupils should be taught to:

- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- multiply and divide numbers mentally drawing upon known facts
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
- recognise and use square numbers and cube numbers, and the notation for squared (2) and cubed (3)
- solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes

- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

### **Number – Fractions (including Decimals and Percentages)**

Pupils should be taught to:

- compare and order fractions whose denominators are all multiples of the same number
- identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths
- recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements  $> 1$  as a mixed number [for example,  $2/5 + 4/5 = 6/5 = 1 \frac{1}{5}$ ]
- add and subtract fractions with the same denominator and denominators that are multiples of the same number
- multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams
- read and write decimal numbers as fractions [for example,  $0.71 = 71/100$ ]
- recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents
- round decimals with two decimal places to the nearest whole number and to one decimal place
- read, write, order and compare numbers with up to three decimal places
- solve problems involving number up to three decimal places
- recognise the per cent symbol (%) and understand that per cent relates to ‘number of parts per hundred’, and write percentages as a fraction with denominator 100, and as a decimal
- solve problems which require knowing percentage and decimal equivalents of  $1/2$ ,  $1/4$ ,  $1/5$ ,  $2/5$ ,  $4/5$  and those fractions with a denominator of a multiple of 10 or 25.

### **Measurement**

Pupils should be taught to:

- convert between different units of metric measure (for example, kilometre and metre; centimetre and metre; centimetre and millimetre; gram and kilogram; litre and millilitre)
- understand and use approximate equivalences between metric units and common imperial units such as inches, pounds and pints
- measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres
- calculate and compare the area of rectangles (including squares), and including using standard units, square centimetres (cm<sup>2</sup>) and square metres (m<sup>2</sup>) and estimate the area of irregular shapes
- estimate volume [for example, using 1 cm<sup>3</sup> blocks to build cuboids (including cubes)] and capacity [for example, using water]
- solve problems involving converting between units of time
- use all four operations to solve problems involving measure [for example, length, mass, volume, money] using decimal notation, including scaling.

### **Geometry – Properties of Shapes**

Pupils should be taught to:

- identify 3-D shapes, including cubes and other cuboids, from 2-D representations
- know angles are measured in degrees: estimate and compare acute, obtuse and reflex angles
- draw given angles, and measure them in degrees
- identify: angles at a point and one whole turn (total 360), angles at a point on a straight line and  $1/2$  a turn (total 180) and other multiples of 90.

- use the properties of rectangles to deduce related facts and find missing lengths and angles
- distinguish between regular and irregular polygons based on reasoning about equal sides and angles.

### **Geometry – Position and Direction**

Pupils should be taught to:

- identify, describe and represent the position of a shape following a reflection or translation, using the appropriate language, and know that the shape has not changed.

### **Statistics**

Pupils should be taught to:

- solve comparison, sum and difference problems using information presented in a line graph
- complete, read and interpret information in tables, including timetables.

### **Year 6 Programme of Study**

#### **Number – Number and Place Value**

Pupils should be taught to:

- read, write, order and compare numbers up to 10 000 000 and determine the value of each digit
- round any whole number to a required degree of accuracy
- use negative numbers in context, and calculate intervals across zero
- solve number and practical problems that involve all of the above.

#### **Number – Addition, Subtraction, Multiplication and Division**

Pupils should be taught to:

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- perform mental calculations, including with mixed operations and large numbers
- identify common factors, common multiples and prime numbers
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

## Number – Fractions (including Decimals and Percentages)

Pupils should be taught to:

- use common factors to simplify fractions; use common multiples to express fractions in the same denomination
- compare and order fractions, including fractions  $> 1$
- add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- multiply simple pairs of proper fractions, writing the answer in its simplest form [for example,  $1/4 \times 1/2 = 1/8$ ]
- divide proper fractions by whole numbers [for example,  $1/3 \div 2 = 1/6$ ]
- associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example,  $3/8$ ]
- identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places
- multiply one-digit numbers with up to two decimal places by whole numbers
- use written division methods in cases where the answer has up to two decimal places
- solve problems which require answers to be rounded to specified degrees of accuracy
- recall and use equivalences between simple fractions, decimals and percentages, including in different contexts.

## Ratio and Proportion

Pupils should be taught to:

- solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts
- solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison
- solve problems involving similar shapes where the scale factor is known or can be found
- solve problems involving unequal sharing and grouping using knowledge of fractions and multiples.

## Algebra

Pupils should be taught to:

- use simple formulae
- generate and describe linear number sequences
- express missing number problems algebraically
- find pairs of numbers that satisfy an equation with two unknowns
- enumerate possibilities of combinations of two variables.

## Measurement

Pupils should be taught to:

- solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate
- use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation up to three decimal places
- convert between miles and kilometres
- recognise that shapes with the same areas can have different perimeters and vice versa

- recognise when it is possible to use formulae for area and volume of shapes
- calculate the area of parallelograms and triangles
- calculate, estimate and compare volume of cubes and cuboids using standard units, including cubic centimetres (cm<sup>3</sup>) and cubic metres (m<sup>3</sup>), and extending to other units [for example, mm<sup>3</sup> and km<sup>3</sup>].

### **Geometry – Properties of Shapes**

Pupils should be taught to:

- draw 2-D shapes using given dimensions and angles
- recognise, describe and build simple 3-D shapes, including making nets
- compare and classify geometric shapes based on their properties and sizes and find unknown angles in any triangles, quadrilaterals, and regular polygons
- illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius
- recognise angles where they meet at a point, are on a straight line, or are vertically opposite, and find missing angles.

### **Geometry – Position and Direction**

Pupils should be taught to:

- describe positions on the full coordinate grid (all four quadrants)
- draw and translate simple shapes on the coordinate plane, and reflect them in the axes.

### **Statistics**

Pupils should be taught to:

- interpret and construct pie charts and line graphs and use these to solve problems
- calculate and interpret the mean as an average.

### **Key Stage 3 (Years 7,8 and 9)**

#### **Working Mathematically**

Through the mathematics content, pupils should be taught to:

#### **Develop fluency**

- consolidate their numerical and mathematical capability from key stage 2 and extend their understanding of the number system and place value to include decimals, fractions, powers and roots
- select and use appropriate calculation strategies to solve increasingly complex problems
- use algebra to generalise the structure of arithmetic, including to formulate mathematical relationships
- substitute values in expressions, rearrange and simplify expressions, and solve equations
- move freely between different numerical, algebraic, graphical and diagrammatic representations [for example, equivalent fractions, fractions and decimals, and equations and graphs]
- develop algebraic and graphical fluency, including understanding linear and simple quadratic functions
- use language and properties precisely to analyse numbers, algebraic expressions, 2-D and 3-D shapes, probability and statistics.

## Reason Mathematically

- extend their understanding of the number system; make connections between number relationships, and their algebraic and graphical representations
- extend and formalise their knowledge of ratio and proportion in working with measures and geometry, and in formulating proportional relations algebraically
- identify variables and express relations between variables algebraically and graphically
- make and test conjectures about patterns and relationships; look for proofs or counter-examples
- begin to reason deductively in geometry, number and algebra, including using geometrical constructions
- interpret when the structure of a numerical problem requires additive, multiplicative or proportional reasoning
- explore what can and cannot be inferred in statistical and probabilistic settings, and begin to express their arguments formally.

## Solve Problems

- develop their mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems
- develop their use of formal mathematical knowledge to interpret and solve problems, including in financial mathematics
- begin to model situations mathematically and express the results using a range of formal mathematical representations
- select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems.

## Subject Content *(Not divided up into specific year group programmes of study)*

### Number

Pupils should be taught to:

- understand and use place value for decimals, measures and integers of any size
- order positive and negative integers, decimals and fractions; use the number line as a model for ordering of the real numbers; use the symbols =, ≠, <, >, ≤, ≥ • use the concepts and vocabulary of prime numbers, factors (or divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation property
- use the four operations, including formal written methods, applied to integers, decimals, proper and improper fractions, and mixed numbers, all both positive and negative
- use conventional notation for the priority of operations, including brackets, powers, roots and reciprocals
- recognise and use relationships between operations including inverse operations
- use integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5 and distinguish between exact representations of roots and their decimal approximations
- interpret and compare numbers in standard form  $A \times 10^n$   $1 \leq A < 10$ , where  $n$  is a positive or negative integer or zero
- work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and  $\frac{7}{2}$  or 0.375 and  $\frac{3}{8}$ )
- define percentage as 'number of parts per hundred', interpret percentages and percentage changes as a fraction or a decimal, interpret these multiplicatively, express one quantity as a percentage of another, compare two quantities using percentages, and work with percentages greater than 100%
- interpret fractions and percentages as operators
- use standard units of mass, length, time, money and other measures, including with decimal quantities
- round numbers and measures to an appropriate degree of accuracy [for example, to a number of decimal places or significant figures]

- use approximation through rounding to estimate answers and calculate possible resulting errors expressed using inequality notation  $a < x \leq b$
- use a calculator and other technologies to calculate results accurately and then interpret them appropriately
- appreciate the infinite nature of the sets of integers, real and rational numbers.

## Algebra

Pupils should be taught to:

- use and interpret algebraic notation, including:  
 $ab$  in place of  $a \times b$ ,  $3y$  in place of  $y + y + y$  and  $3 \times y$ ,  $a^2$  in place of  $a \times a$ ,  $a^3$  in place of  $a \times a \times a$ ;  $a2b$  in place of  $a \times a \times b$ ,  $a/b$  in place of  $a \div b$
- coefficients written as fractions rather than as decimals
- brackets
- substitute numerical values into formulae and expressions, including scientific formulae
- understand and use the concepts and vocabulary of expressions, equations, inequalities, terms and factors
- simplify and manipulate algebraic expressions to maintain equivalence by:
  - collecting like terms
  - multiplying a single term over a bracket
  - taking out common factors
  - expanding products of two or more binomials
- understand and use standard mathematical formulae; rearrange formulae to change the subject
- model situations or procedures by translating them into algebraic expressions or formulae and by using graphs
- use algebraic methods to solve linear equations in one variable (including all forms that require rearrangement)
- work with coordinates in all four quadrants
- recognise, sketch and produce graphs of linear and quadratic functions of one variable with appropriate scaling, using equations in  $x$  and  $y$  and the Cartesian plane
- interpret mathematical relationships both algebraically and graphically
- reduce a given linear equation in two variables to the standard form  $y = mx + c$ ; calculate and interpret gradients and intercepts of graphs of such linear equations numerically, graphically and algebraically
- use linear and quadratic graphs to estimate values of  $y$  for given values of  $x$  and vice versa and to find approximate solutions of simultaneous linear equations
- find approximate solutions to contextual problems from given graphs of a variety of functions, including piece-wise linear, exponential and reciprocal graphs
- generate terms of a sequence from either a term-to-term or a position-to-term rule
- recognise arithmetic sequences and find the  $n$ th term
- recognise geometric sequences and appreciate other sequences that arise.

## Ratio, Proportion and Rates of Change

Pupils should be taught to:

- change freely between related standard units [for example time, length, area, volume/capacity, mass]
- use scale factors, scale diagrams and maps
- express one quantity as a fraction of another, where the fraction is less than 1 and greater than 1
- use ratio notation, including reduction to simplest form
- divide a given quantity into two parts in a given part:part or part:whole ratio; express the division of a quantity into two parts as a ratio

- understand that a multiplicative relationship between two quantities can be expressed as a ratio or a fraction
- relate the language of ratios and the associated calculations to the arithmetic of fractions and to linear functions
- solve problems involving percentage change, including: percentage increase, decrease and original value problems and simple interest in financial mathematics
- solve problems involving direct and inverse proportion, including graphical and algebraic representations
- use compound units such as speed, unit pricing and density to solve problems.

## **Geometry and Measures**

Pupils should be taught to:

- derive and apply formulae to calculate and solve problems involving: perimeter and area of triangles, parallelograms, trapezia, volume of cuboids (including cubes) and other prisms (including cylinders)
- calculate and solve problems involving: perimeters of 2-D shapes (including circles), areas of circles and composite shapes
- draw and measure line segments and angles in geometric figures, including interpreting scale drawings
- derive and use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle); recognise and use the perpendicular distance from a point to a line as the shortest distance to the line
- describe, sketch and draw using conventional terms and notations: points, lines, parallel lines, perpendicular lines, right angles, regular polygons, and other polygons that are reflectively and rotationally symmetric
- use the standard conventions for labelling the sides and angles of triangle ABC, and know and use the criteria for congruence of triangles
- derive and illustrate properties of triangles, quadrilaterals, circles, and other plane figures [for example, equal lengths and angles] using appropriate language and technologies
- identify properties of, and describe the results of, translations, rotations and reflections applied to given figures
- identify and construct congruent triangles, and construct similar shapes by enlargement, with and without coordinate grids
- apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles
- understand and use the relationship between parallel lines and alternate and corresponding angles
- derive and use the sum of angles in a triangle and use it to deduce the angle sum in any polygon, and to derive properties of regular polygons
- apply angle facts, triangle congruence, similarity and properties of quadrilaterals to derive results about angles and sides, including Pythagoras' Theorem, and use known results to obtain simple proofs
- use Pythagoras' Theorem and trigonometric ratios in similar triangles to solve problems involving right-angled triangles
- use the properties of faces, surfaces, edges and vertices of cubes, cuboids, prisms, cylinders, pyramids, cones and spheres to solve problems in 3-D
- interpret mathematical relationships both algebraically and geometrically.

## **Probability**

Pupils should be taught to:

- record, describe and analyse the frequency of outcomes of simple probability experiments involving randomness, fairness, equally and unequally likely outcomes, using appropriate language and the 0-1 probability scale
- understand that the probabilities of all possible outcomes sum to 1
- enumerate sets and unions/intersections of sets systematically, using tables, grids and Venn diagrams

- generate theoretical sample spaces for single and combined events with equally likely, mutually exclusive outcomes and use these to calculate theoretical probabilities.

## **Statistics**

Pupils should be taught to:

- describe, interpret and compare observed distributions of a single variable through: appropriate graphical representation involving discrete, continuous and grouped data; and appropriate measures of central tendency (mean, mode, median) and spread (range, consideration of outliers)
- construct and interpret appropriate tables, charts, and diagrams, including frequency tables, bar charts, pie charts, and pictograms for categorical data, and vertical line (or bar) charts for ungrouped and grouped numerical data
- describe simple mathematical relationships between two variables (bivariate data) in observational and experimental contexts and illustrate using scatter graphs